(7 pages) **Reg. No. :** ....

Code No. : 6368 **Sub. Code : ZMAM 15**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

First Semester

Mathematics – Core

ORDINARY DIFFERENTIAL EQUATION

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Which of the following second order non-homogenous linear differential equation?

(a) $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y^2 = R(x)$

(b) $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y^2 = 0$

(c) $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = R(x)$

(d) $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$

2. Any linear combination of two solution of the homogeneous equation is ————

(a) Not a solution (b) Solution

(c) Linear (d) Non-linear

3. If $f(x)$ and $g(x)$ are analytic at x_0, then ———— is analytic.

(a) $f(x) + g(x)$ (b) $f(x)g(x)$

(c) $\frac{f(x)}{g(x)}$ if $g(x_0) \neq 0$ (d) All the above

4. The series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ ————

(a) converges for all x

(b) converges only at $|x| < 1$

(c) diverges at $|x| > 1$

(d) diverges for all $x \neq 0$

5. Which of the following series is Frobenius series?

(a) $y = a_0 + a_1x^m + a_2x^{(m+1)} + \ldots$

(b) $y = a_0x^m + a_1x^{m+1} + a_2x^{m+2} + \ldots$

(c) $y = x^m(a_0 + a_2 + a_1 + \ldots)$

(d) $y = x^m(a_0x + a_1x^2 + a_2x^3 + \ldots)$

6. If the function $(x - x_0)P(x)$ and $(x - x_0)^2Q(x)$ are not analytic then singular point x_0 is said to be ————

(a) Irregular (b) Regular

(c) Singular (d) Non-Singular

7. $J_{\frac{-1}{2}}(x) =$

(a) $\sqrt{\frac{2}{\pi x}}\cos x$ (b) $\sqrt{\frac{1}{\pi x}}\cos x$

(c) $\sqrt{\frac{2}{\pi x}}\sin x$ (d) $\sqrt{\frac{1}{\pi x}}\sin x$

8. $\Gamma(p) =$

(a) $\Gamma(p+1)$ (b) $\Gamma(p-1)$

(c) $\frac{\Gamma(p+1)}{p}$ (d) $p\Gamma(p+1)$

9. The value of $\begin{vmatrix} e^{3t} & e^{2t} \\ e^{3t} & 4e^{2t} \end{vmatrix}$

(a) e^{4t} (b) $3e^{5t}$

(c) $2e^{5t}$ (d) $3e^{4t}$

10. The auxiliary question of $\begin{cases} \frac{dx}{dt} = 3x - 4y \\ \frac{dy}{dt} = x - y \end{cases}$ is

(a) $(m-1)^2 = 0$ (b) $(m+1)^2 = 0$

(c) $m^2 + 2m - 1 = 0$ (d) $m^2 - 2m - 1 = 0$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that if $y_1(x)$ and $y_2(x)$ are any two solution of $y'' + P(x)y' + Q(x)y = 0$, then $c_1y_1(x) + c_2y_2(x)$ is also a solution for any constants c_1 and c_2.

Or

(b) Show that $y = c_1x + c_2x^2$ is the general solution of $x^2y'' - 2xy' + 2y = 0$ on any interval not containing 0, and find the particular solution for which $y(1) = 3$ and $y'(1) = 5$.

12. (a) Find the general solution of $(1 + x^2)y'' + 2xy' - 2y = 0$ in terms of power series in x.

Or

(b) Find the power series for $\frac{1}{(1-x)^2}$ by squaring.

[P.T.O.]

13. (a) Find two independent Frobenius series solutions of $x^2y'' - x^2y' + (x^2 - 2)y = 0$.

Or

(b) Determine the nature of the point $x = 0$ for $y'' + (\sin x)y = 0$.

14. (a) Show that $\frac{d}{dx}[xJ_1(x)] = xJ_0(x)$.

Or

(b) Prove that $\frac{d}{dx}[x^pJ_p(x)] = x^pJ_{p-1}(x)$.

15. (a) Show that $\begin{cases} x = e^{4t} \\ y = e^{4t} \end{cases}$ and $\begin{cases} x = e^{-2t} \\ y = -e^{-2t} \end{cases}$ are solutions of the homogeneous system

$$\begin{cases} \frac{dx}{dt} = x + 3y \\ \frac{dy}{dt} = 3x + y \end{cases}.$$

Or

(b) Show that $\begin{cases} x = 2e^{4t} \\ y = 3e^{4t} \end{cases}$ and $\begin{cases} x = e^{-t} \\ y = -e^{-t} \end{cases}$ are solutions of the homogeneous system

$$\begin{cases} \frac{dx}{dt} = x + 2y \\ \frac{dy}{dt} = 3x + 2y \end{cases}.$$

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b)

16. (a) Show that $y = c_1 \sin x + c_2 \cos x$ is the general solution of $y'' + y = 0$ on any interval, and find the particular solution for which $y(0) = 2$ and $y'(0) = 3$.

Or

(b) Find the differential equation of $y = c_1 \sin kx + c_2 \cos kx$ and $y = c_1 + c_2e^{-2x}$ by eliminating the constants c_1 and c_2.

17. (a) Derive Binomial Series expansion.

Or

(b) Let x_0 be an arbitrary point of the differential equation $y'' + P(x)y' + Q(x)y = 0$, and let a_0 and a_1 be arbitrary constants. Then prove that there exists a unique function $y(x)$ that is analytic at x_0, is a solution of the equation in a certain neighborhood of this point and satisfies the initial conditions $y(x_0) = a_0$ and $y'(x) = a_1$. Furthermore, prove that if the power series expansion of $P(x)$ and $Q(x)$ are valid on an interval $|x - x_0| < R, R > 0$, then the power series expansion of this solution is also valid on the same interval.

18. (a) Verify that the origin is a regular singular point and calculate two independent Frobenius series solution for $2xy'' + (3 - x)y' - y = 0$.

Or

(b) Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} 0 & if\ m \neq n \\ \frac{2}{2n+1} & if\ m = n \end{cases}$, where $P_0(x), P_1(x), P_2(x), \ldots, P_n(x)$ is asequence of orthogonal functions on the interval $-1 \le x \le 1$.

19. (a) Find the value of $\Gamma\left(\frac{1}{2}\right)$.

Or

(b) Prove that

$$\int_0^1 xJ_p(\lambda_m x)J_p(\lambda_n x)dx = \begin{cases} 0 & if \quad m \neq n \\ \frac{1}{2}J_{p+1}(\lambda_n)^2 & if \quad m = n \end{cases}$$

20. (a) Find the General solution of $\begin{cases} \frac{dx}{dt} = 7x + 6y \\ \frac{dy}{dt} = 2x + 6y \end{cases}$.

Or

(b) Find the General solution of $\begin{cases} \frac{dx}{dt} = -4x - y \\ \frac{dy}{dt} = x - 2y \end{cases}$.